RESEARCH FOUNDATION PAPER



A Holistic Approach to Teaching Early Childhood Mathematics

Written by Jie-Qi Chen, Ph.D.

Founder, Early Math Collaborative at the Erikson Institute



Recent research indicates that what children know about mathematics in the early years predicts both their mathematics and reading achievement for years to come.

Duncan et al. 2007

No part of this publication may be reproduced in whole or in part, or stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without written permission of the publisher. For information regarding permission, write to Scholastic Inc., 557 Broadway, New York, NY 10012.

Copyright © 2021 by Scholastic Inc. All rights reserved. Published by Scholastic Inc. Printed in the U.S.A.

DISTINCTIVE FEATURES OF THE PREK ON MY WAY MATH MODULE

A curriculum is more than a collection of activities; it needs to be coherent, focused on important concepts and skills, accommodate a range of learners, and support teachers' implementation. A good early childhood curriculum is designed to not only provide teachers with ideas about what to teach, but also how to teach with respect to the developmental needs of young children and how to teach a particular set of concepts and skills in the content areas. An effective curriculum guides teachers to plan and instruct, at the same time encouraging them to think and explore with adequate support.

The **PreK On My Way Math Module** (*Math Module* hereafter) is inentionally designed to help early childhood practitioners navigate the goals of teaching an early mathematics curriculum. The *Math Module* supports these goals by integrating an important set of features that help early childhood teachers foster understanding, develop skills, and nurture the minds of young mathematicians. The distinct features included in the *Math Module* allow teachers who use the curriculum to:

- 1. Focus on diverse early mathematics content strands
- 2. Attend to cross-content connections in early mathematics
- 3. Define early mathematical learning goals and skills
- 4. Offer rich daily activities and help childrendevelop flexible thinking skills
- 5. Mathematize learning through children's daily routines
- 6. Use multiple representations to foster deep understanding
- 7. Integrate children's literature with early mathematics learning
- 8. Emphasize the use of mathematics language
- 9. Connect math, language, and literacy learning through social-emotional learning
- 10. Make informed decisions and exercise best practices
- 11. Build home-school partnerships

This paper will identify why each of these distinct features is integral to a holistic approach to teaching early childhood mathematics, and describe how the *Math Module* curriculum is designed to support teachers' strong understanding of the "what, how, and why" of teaching early mathematics.

1. FOCUS ON DIVERSE EARLY MATHEMATICS CONTENT STRANDS

The traditional early mathematics curriculum emphasizes numbers and shapes, considering these two topics to be the whole of mathematics that is appropriate for young children. We know now that **mathematics should be considered as a plural noun**, meaning that math incorporates many different content areas with many different types of calculations and formulas. To build foundations in mathematics learning, young children need to be exposed to a variety of mathematics topics in addition to numbers and shapes, such as spatial thinking, sets, patterns, and measurement. When we equate early math with only one or two topics, we deprive young children of opportunities to build a strong mathematics foundation for further learning and development.

The *Math Module* consists of four interrelated early math content areas: **numbers and operations, geometry and spatial sense, measurement, and classification and patterns**. Each area explores a range of mathematical concepts and skills. Numbers and operations, for example, include skills such as one-to-one correspondence, counting, subitizing, cardinality, ordinality, adding and taking away, composing and decomposing, and fair share. In the case of measurement, children learn to identify, describe, and compare a variety of measurable attributes, including height, length, capacity, weight, and time. By providing a wide array of early math content areas and connecting them through rich classroom experiences, the *Math Module* helps young children develop rich mathematics concepts and understanding.



Interrelated Early Math Content Areas in the Math Module

2. ATTEND TO CROSS-CONTENT CONNECTIONS IN EARLY MATHEMATICS

To learn mathematics, young children must be actively involved with activities in different content strands. Equally important is their knowledge of the connections between these content strands. Defined by the National Council of Teachers of Mathematics, the cross-content topics include **unitizing, relating and organizing, decomposing and composing, and looking for patterns and structures** (NCTM 2000). These overarching concepts connect mathematical ideas, procedures, and problem situations into a unified body of knowledge. They also act as **"vehicles for children to deepen, extend, elaborate, and refine their thinking"** and **"explore ideas and lines of reasoning"** (NRC 2009).

The *Math Module* defines each of the cross-content topics and invites children to engage in Higher-Order Mathematical Thinking Skills as they unitize, compare, decompose and compose, and look for patterns and structure across math learning experiences. In so doing, the *Math Module* supports young children as they develop a range of abilities essential to mathematical understanding far beyond their preschool years, including:

- 1. Unitizing—the ability to find or make a unit, such as a **repeating pattern unit** of AB, a **measuring unit** of block lengths or inches, and a **counting unit** of either one shoe or pair of shoes
- Relating and organizing—the ability to compare items to order them, such as identifying objects by size and ordering them when children engage in a measurement activity or comparing and ordering a set of objects by quantity when counting
- 3. Decomposing and composing—the ability to take apart and put together parts into a whole, such as two triangles to make a square in **spatial reasoning** and put 3 and 2 together to make 5 in **number operations**
- 4. Looking for patterns and structures the ability to notice rules that always apply. For example, a given collection of items has a consistent quantity regardless of the way it is counted, and triangles can differ in size and orientation but are still triangles.

Higher-Order Mathematical Thinking Skills Across Math Content Areas in the Math Module



3. DEFINE EARLY MATHEMATICAL LEARNING GOALS AND SKILLS

Learning goals address the question of **what children ought to know or be able to do** by the end of a given period of instruction. Clearly defined learning goals help teachers prepare the teaching experience, monitor the teaching process, and evaluate teaching effectiveness. Learning goals should not place limits on what teachers teach. Rather, they should provide benchmarks for teachers to gauge children's progress toward foundational mathematics learning in the early years of schooling.

The age-specific learning goals in the Math Module are presented in four ways:

- As a prekindergarten curriculum, the learning goals focus on four-year-olds. Keenly aware of the developmental variations in young children, the learning goals also extend to three-year-olds the children who require more support—as well as five-year-olds—the children who are ready for more stimulation or challenges.
- 2. Regardless of age, the learning goals in the *Math Module* are defined in terms of **specific skills** in each of the four content strands discussed in the previous section, thus providing teachers with actionable items for instruction.
- 3. The development of **later skills is clearly connected to the previous ones**. For example, a child's ability to count aloud 1–10 items with one-to-one correspondence builds on her skills of one-to-one correspondence and knowing number words from 1 to 10 in order.
- 4. All **learning goals are aligned with national standards** such as the Head Start Early Learning Outcomes Framework, Gold, and Common Core State Standards, as well as a set of **state early learning goals**, including those from Texas, New York, California, and New Jersey.

| 3-Year-Olds: Requiring More Support | 4-Year-Olds: Typically Developing | 5-Year-Olds: Ready for More Challenges |
|---|--|--|
| Identify individual items and understand that items can be counted. | Understand that items and amounts can be counted. | Understand the relationship between numbers and quantities; connect counting to cardinality. |
| Recite number words from 1–10 in order, understanding that each counting word is a separate word. | Recite number words from 1 to 30 in order. | Count to 100 by ones and tens. |
| Count four objects with one-to- one correspondence. | Count aloud 1–10 items with one- to-one correspondence. | Count objects by naming the number in the standard order, pairing each object with one and only one number name, and naming each number with one and only one object. |

Age-Specific Learning Goals

4. OFFER RICH DAILY ACTIVITIES AND HELP CHILDREN DEVELOP FLEXIBLE THINKING SKILLS

Traditionally, math is a place where there is "one right answer." The risk is that children can easily fall into a habit of thinking that the goal of math learning is solely to get all the answers right, and that there is only one way to get there. Developmental math, on the contrary, fosters a growth mindset by emphasizing more than one way to solve problems. Rich materials and activities provide children with multiple ways to show mastery of concepts. Children who have more than one way to solve a problem develop flexible thinking skills, and **when habits of flexible thinking start to hatch, they can be generalized to other areas of learning and life** (Brownell et al. 2018, Magalhães et al. 2020).

The *Math Module* includes a range of small-group activities that support the development of children's flexible thinking skills. For example, children's **number sense and understanding of quantities are introduced and reinforced in multiple ways** illustrated below.

- **1. Act It Out:** Children practice hands-on math skills such as counting by acting out story scenarios to better understand the value of quantities and number.
- **2. Matching Game:** Children match different sets to show that a quantity remains the same even if materials and arrangements change.
- **3. Number Line, 5-Frame, 10-Frame, and Rekenrek:** Children use these tools to strengthen their number sense as they count, add, take away, and compare.
- **4. Give Me____:** Children practice counting out a certain number of objects from a larger group using play scenarios. (e.g., In the market: "I would like to buy 6 of those 10 oranges." Children count out 6 of their 10 orange Connecting Cubes..
- 5. Behind the Curtain: Children expand from concrete to abstract thinking as they "imagine" what is behind the curtain. (e.g., Three counting bears go behind the curtain. Two more bears join them. How many bears are there behind the curtain altogether?.
- **6. Spot the Mistake:** Children finding the mistake in a counting sequence or pattern is a fun and effective way to flip the learning, similar to "oddity tasks" in early literacy. (e.g., Which word does not rhyme?.
- **7. Graph It!:** Children use graphs to share information. (e.g., How many children like apples, bananas, or melons?)

Multiple Ways of Learning and Understanding Numbers And Quantities

How Many Dots?

Gather all of the dot Number Cards showing five or fewer dots. Show children the cards one at a time for two seconds and have them call out how many dots they see.



Sing a Counting Song

Sing a counting song with children, like "Five Little Ducks." Use your fingers to show the amount of ducks each time you sing a number.

Act It Out 🕄

Show children a magnetic number from one to five, but don't say the number aloud. Ask children to jump, wiggle, or touch their toes that many times while counting aloud.

5. MATHEMATIZE LEARNING THROUGH CHILDREN'S DAILY ROUTINES

Young children are inherently curious and equipped with basic capacities and dispositions to make sense of the world around them. When given the opportunity, children naturally explore a variety of mathematical ideas during play, including comparison, estimation, patterns, and spatial relationships. However, **play alone does not necessarily lead children to construct mathematical meanings from their experiences**. Similarly, mathematics exists everywhere in the natural and human-made worlds, such as symmetry in certain plants, the natural pattern of four seasons, geometric shapes in building design, and different sizes of shoes. Yet these **embedded mathematical concepts are not always easy for young children to grasp**. It is our job as adults to identify and highlight the mathematics that children experience to help them become aware of these experiences as mathematics. This is what we call **mathematizing**.

The *Math Module* aims to ensure that math learning and problem-solving situations are built into the very fabric of children's lives. **The entire math curriculum is organized through eight themes**, such as "Me and You," "Our Community," and "Our Earth." The themed topics are familiar to children and relate to their personal experiences. In the *Math Module*, **mathematics teaching and learning is not limited to math lesson time; it also takes place throughout the day in learning center time and daily routines**. Children acquire number knowledge and develop spatial reasoning through teacher-directed math lessons as well as self-selected activities during center time, transition time, snack time, and outdoor physical exercise. Teachers mathematize the situation by inviting children to **notice** ("I see how you match each fork with a plate."), and asking them to **predict** ("What do you think will happen if I ask you to count the sidewalk cracks in a different way?"), and they **model** ways to problem-solve ("Look what happens when I use my shoes, not yours, to measure this rug.").



6. USE MULTIPLE REPRESENTATIONS TO FOSTER DEEP UNDERSTANDING

Research indicates that when new ideas are presented in many different ways, learners' brains form more connections between the new ideas and past experiences. Of significant importance to effective early math learning and teaching is the **interaction of actions, images, and symbols** (Mitchell 2013). Specifically, math actions include such behavior as manipulating, gesturing, and dramatizing. Math images might consist of pictures, graphs, and mental images. Symbols focus mainly on math language and simple math symbols, such as numerals, plus (+), minus (-), and equal (=) signs. Diverse modes of representation in mathematics learning activate children's multiple senses as well as their information-processing capacities and therefore build stronger connections among different concepts and ideas (Chen 2017).

In the *Math Module* children enjoy books, activities, and purposeful play with math actions, images, and symbols.

- Actions: Children actively gesture and dramatize quantities, comparisons, patterns, and operations through small-group activities, interactive read-alouds, purposeful play, and music and movement opportunities.
- **Images:** Children see and create both **concrete and mental images** through experiences with high-quality read-alouds, number cards, photo cards, and hands-on activities with drawing, writing, graphing, and arts and crafts.
- **Symbols:** Children learn to use **math language and written symbols** to describe the math they witness and create in everyday life.

What is more, the actions, images, and symbols in the *Math Module* work in tandem to reinforce different math concepts and skills. For example, to support the understanding of cardinal sense of number 5, the *Math Module* invites children to jump, wiggle, or touch their toes five times, use different manipulatives such as cubes, counters, buttons to represent 5; read aloud picture books like *5 Little Monkeys*; draw pictures, dots, and tally marks to represent 5; and use the number card "5" to discuss the meaning of how many. Children's math learning is significantly augmented through their **eyes, ears, hands, mental images, and language**.



7. INTEGRATE CHILDREN'S LITERATURE WITH EARLY MATHEMATICS LEARNING

All children love picture books. There are many great picture books, stories, and nursery rhymes that are very dear to our hearts. When we **integrate children's literature with early math learning, we bring math problems to life**. That is, we provide context to help children learn math concepts and math-related vocabulary, show them how math is used in a variety of situations, and lead rich math discussions among children (Hong 1996). Stories also provide an engaging introduction to new math content and greater access to math for children who enjoy literacy and reading, which helps reduce math anxiety. A picture book further creates a shared experience that can be discussed and analyzed by children and the teacher, time and again. Such repeated experiences give children ample opportunities to revisit the math situation from many different angles and understanding levels.

For more than 100 years, Scholastic has produced and distributed high-quality children's books. In the *Math Module*, **quality children's literature—including picture books, informational texts, and nursery rhymes—is used to introduce mathematical concepts**. For example, the *Math Module's Creature Features* by Penelope Arlon and Tory Gordon-Harris is an informational text that introduces children to animals and their fascinating features. The book invites children to identify animal attributes and count animal features. It is important to note that (1) a book read-aloud is scheduled daily during the large-group time to ensure a shared experience among all children, and (2) all of the books used in the *Math Module* have a Spanish version to capitalize on children's home language skills in mathematical learning.



8. EMPHASIZE THE USE OF MATHEMATICS LANGUAGE

Manipulatives are necessary but alone are not sufficient for the development of young children's mathematical knowledge. Equally important for early mathematical learning is the use of mathematical language—language that relates mathematical concepts and facilitates mathematical thinking. Research indicates that the more preschool teachers talk about math, the stronger children perform in math learning. It is for this reason that **adult math talk matters**. Language input from adults helps young children articulate, elaborate, and consolidate the math concepts they are developing.

The *Math Module* promotes and reinforces the use of math language for both teachers and children. Math vocabulary cards, for example, are used throughout the module to illustrate story words and math language. In the case of math vocabulary card "sort," when the term is initially introduced, the teacher first asks children to repeat after her, then defines and describes the term, and finally invites children to act it out with her. In the follow-up activity, children are encouraged to repeat, rephrase, or expand the term through discussion. Through this intentional process, children learn math vocabulary with understanding and through engaging games. In the *Math Module*, open-ended questions from teachers such as "why" and "how do you know" are especially prominent (see the illustration). Using the *Math Module*, teachers work with young children and involve them in **sharing math strategies**, explaining math thinking, and engaging in mathematical discourse with them as well as peers.





Math Vocabulary Card

Sample Open-Ended Mathematical Questions

- Where do you see this (mathematical idea) in our classroom?
- How did you figure this out (this mathematical idea)?
- What happens when I break this apart/put these together?
- Do you see a pattern here? What is it?
- How do you know this is taller/ wider/skinnier than the other one?
- Can you think of another way to sort these cubes?

9. CONNECT MATH, LANGUAGE, AND LITERACY LEARNING THROUGH SOCIAL-EMOTIONAL LEARNING

There are many parallels between math and literacy learning. For example, children learn the letters of the alphabet and come to understand and use them as symbols of spoken language just as they learn numbers and come to understand them as symbols of quantities. Children learn to order details in a story to support comprehension just as they learn to order items and quantities in order to solve real-world math problems. Being aware of these similarities can help alleviate math anxiety and facilitate understanding of how valuable early math is to children's overall learning (Hong 1996 Negen and Sarnecka 2012).

In the Math Module, math, language, and literacy learning are intertwined in many ways, including through the weekly concept question, children's books, number cards, thematic magazines, math talks, and daily writing. Distinctive to this connection is that it is grounded in and supported by Mind Builders, which promote social, emotional, executive function, and motivation and creativity skills. For example, one of the Mind Builder skills is thinking symbolically or having the ability to use one thing to stand for something else. In the Math Module, children develop this skill through connecting concrete manipulatives with numerals and the concept of cardinality, or using a variety of tools such as counting bears, tally marks, pictures, or dots to represent the same number.

| Example Motivation and Creativity Mind Builder SKILL: Thinking symbolically DEFINITION: Use one thing to stand for something else | | | |
|---|--|--|--|
| | | | |

10. MAKE INFORMED DECISIONS AND EXERCISE BEST PRACTICES

A curriculum provides a road map to a destination. **Curriculum implementation is a knowledge reconstruction process** where a teacher needs to translate the curricular text to a particular teaching situation. This process is complex and requires adaptation. Early childhood classrooms vary in terms of space and materials, as well as the number of children and their developmental levels. Instead of using rigid step-by-step procedures and instructions, which can impede teachers' imaginations, a good curriculum empowers teachers by providing them with the rationale behind a particular instructional practice and supports flexible implementation.

Of critical importance to the *Math Module's* philosophy is that teachers are the ones best equipped to make decisions about the children in their classrooms. Consistent with this philosophy, the *Math Module* describes learning goals and skills for typically developing four-year-olds, as well as for children who proceed at different developmental paces. The *Math Module* also suggests various options for implementing the activities, including different grouping possibilities, a range of playful activities, and varied modes of learning. Last but not least, the *Math Module* provides child development tips to articulate the research-based evidence behind particular math concepts or skills. **The Math Module does not believe that one size fits all. It provides teachers with tools and encourages them to make choices and adaptations based on their observations of children.**



Large Group Card: Child Development Mind Builder Tip

11. BUILD HOME-SCHOOL PARTNERSHIPS

Optimal learning occurs when families are involved in the child's learning process. Most families are interested in working with their children in mathematics learning and development. However, what constitutes developmentally appropriate mathematics learning? What does teaching mathematics entail? Are worksheets a good way to foster the child's mathematical understanding of concepts or acquisition of skills? How can home activities such as cooking, laundry, and shopping be mathematized? Families are not always clear about answers to these questions. Home-school partnerships take many forms. One way to help families understand what quality early mathematics experiences look like is to provide them with materials and examples that support such experiences in the home or outside of school.

The *Math Module* provides **bilingual family engagement resources** to bridge learning at school and at home with conversation starters and fun activities. Weekly family bulletins include an opportunity for children to create a book at home that relates to the concept they are exploring at school. The bulletin also reminds families that they can connect to the digital Song Collection and the Little eReaders, both of which relate to the theme that children are exploring and provide opportunities to connect experiences and learning at home and at school. Chat Bands are conversation starter bracelets that children can wear home each week to encourage continued concept exploration and conversation between school and home.



CONCLUSION

Young children are natural mathematicians. They are born with mathematical proclivities that support their natural curiosity about the world around them. Many of the central questions of childhood are mathematical in nature, such as *How many? Who has more? Is it fair? Am I growing bigger?* From the first years of life, children can learn math and develop their interest in math.

Early mathematics supports symbolic development and appears to be a core component of cognition. Numerous studies in the field of mathematical cognition indicate that humans are endowed with a core capacity to understand varied mathematical properties such as quantity, magnitude, spatial-temporal relationships, sequence, and regularity (Baillargeon and Graber 1987, Dehaene 2011, Hynes-Berry, Chen, and Abel in press, Mix, Huttenlocher, and Levine 2002, NRC 2009, Wang and Feigenson 2019, Xu 2003). Mathematical education develops children's logical reasoning, analytical skills, and problem-solving abilities, all of which are critical in any learning experience. Confirming its importance in development, recent research indicates that what children know about **mathematics in the early years predicts both their mathematics and reading achievement for years to come** (Duncan et al. 2007, Krajewski and Schneider 2009, Starr, Libertus, and Brannon 2013). The preschool years are critical for mathematics learning and development.

The PreK On My Way Math Module, with its 11 integrated features, empowers teachers to understand the "what, how, and why" of teaching early mathematics. It is a powerful program designed to help teachers and families work with children to build strong foundational mathematics knowledge and advance early mathematical development in the context of productive play.

ABOUT THE AUTHOR

Jie-Qi Chen, Ph.D.

Dr. Chen is Barbara T. Bowman Professor of Early Childhood Education at Erikson Institute. As the founder of Erikson's Early Math Collaborative, she leads the effort to transform the teaching and learning of early mathematics from the ground up and empower teachers to focus on foundational math learning and thinking. Dr. Chen has inspired the play-based mathematical learning in PreK On My Way, as well as its ongoing professional learning supports for educators.

REFERENCES

- Baillargeon, R., and Graber, M. (1987). Where's the rabbit? 5.5-month-old infants' representation of the height of a hidden object. *Cognitive Development*, Vol. 2(4), 375–392.
- Brownell, J. and Berry, H., with Baroody, A. J. (2018). Pathways to basic combination fluency in the primary grades. In J. McCray, J. Q. Chen, and J. Sorkin (Eds.). *Growing Mathematical Minds: Conversations between Developmental Psychologists and Classroom Teachers*. New York: Routledge.
- Chen, J. Q. (2017). Open mindset in the education of young children. In P. Erath, F-M. Konrad, and M. Rossa (Eds.), *The Kindergarten as an Educational Institution*. Bad Heibrunn, Germany: Klinkhardt.
- Dehaene, S. (2011). The Number Sense: How the Mind Creates Mathematics. Oxford: Oxford University Press.
- Duncan, G. J., et al. (2007). School readiness and later achievement. Developmental Psychology, Vol. 43(6).
- Hong, H. (1996). Effects of mathematics learning through children's literature on math achievement and dispositional outcomes. *Early Childhood Research Quarterly*, Vol. 11(4).
- Hynes-Berry, M., Chen, J. Q., and Abel, B. (in press). *Precursor Math Concepts: The Wonder of Mathematical Worlds with Infants and Toddlers*. New York: Teachers College Press.
- Krajewski, K., and Schneider, W. (2009). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, Vol 19(6).
- Magalhães, S., et al. (2020). Executive functions predict literacy and mathematics achievements: The unique contribution of cognitive flexibility in grades 2, 4, and 6. *A Journal on Normal and Abnormal Development in Childhood and Adolescence*, Vol. 26(7).
- Mitchell, N. (2013). Embodied cognition: What it means to know and do mathematics. National Council Teachers Mathematics Annual Conference Research Presentation, Denver.
- Mix, K. S., Huttenlocher, J., and Levine, S. C. (2002). *Quantitative Development in Infancy and Early Childhood.* Oxford: Oxford University Press.
- National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics.
- National Research Council. (2009). *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity.* Washington, DC: National Academies Press.
- Negen, J., and Sarnecka, B. (2012). Number-concept acquisition and general vocabulary development. *Child Development*, Vol. 83.
- Starr, A., Libertus, M. E., and Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences*, Vol. 110(45).
- Wang, J., and Feigenson, L. (2019). Infants recognize counting as numerically relevant. *Developmental Science*, Vol. 22(6).
- Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. Cognition, Vol. 89(1).



scholastic.com/prekonmyway



Copyright © 2021 by Scholastic Inc